

Application Notes

Magnetics

6

Determining L_{\min} for
Buck/Boost Converters

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DETERMINING L_{MIN} FOR BUCK AND BOOST TYPE CONVERTERS

FUNDAMENTAL CONCEPTS

DEFINITION OF INDUCTANCE: When current flows through a coil, a magnetic field is established around the coil. The magnetic field can be expressed in terms of flux lines and is symbolized by the Greek letter ϕ with the unit of measurement in Webers (one Weber equals 10^8 flux lines). A flux linkage is one flux line encircling the circuit current once. Thus, a coil consisting of N turns would (neglecting leakage flux effects) have $N\phi$ flux linkages for each ampere of current through the coil. Inductance is, by definition, the number of flux linkages per ampere of current. Putting this in the form of an equation gives:

$$L = N\phi/i \quad (1)$$

Where L is the inductance in henrys, N is the number of turns, ϕ is the number of magnetic flux lines in webers, and i is the current in amperes.

INDUCTOR VOLTAGE: Faraday's law states that the induced voltage, e , across an inductor is equal to the time rate of change of magnetic flux linkages, or:

$$e = d(N\phi)/dt = Nd\phi/dt \quad (2)$$

Combining equations 1 and 2 gives

$$e = Ldi/dt \quad (\text{Lenz's Law}) \quad (3)$$

where e is in volts, and t is in seconds. This equation makes the assumption that the magnetic properties of the surrounding medium (primarily the magnetic core) are not functions of the current or physical environment, and that the properties of the inductor do not change with time. Actually, L does vary with

magnetizing force, which in turn is a function of the current through the coil. However, for the purposes of calculating L_{MIN} , the effect is not significant if operation is constrained to the nearly linear portion of the B-H curve as explained in the next few sections.

B-H (HYSTERESIS EFFECTS): Fig. 1 shows the familiar B-H hysteresis loop of a typical ferromagnetic core material. When a magnetizing force, H , (proportional to NI) is first applied, the flux density, B , changes in a fairly linear fashion along the path o-a. Further increase of H , however, fails to increase B at the same rate because the material saturates, i.e., the material becomes fully magnetized. This is indicated in Fig. 1 as B_{SAT} .

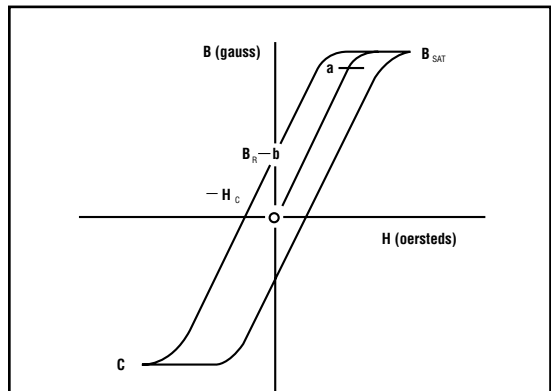


FIGURE 1. B-H Loop

The ratio of B to H is the permeability, μ , of the core material; and the slope of line o-a is referred to as the initial permeability. When the magnetizing force is returned to zero, the flux density does not retrace along a-o, but instead, follows a different path to point b, which is called the residual flux density, B_R . This occurs because the core material inherently has memory which retains magnetic information. At this point a negative value of H , called the coercive force, H_C , must be applied to return B to zero. If the magnetizing force is increased further in the negative direction, the flux density will move to point c.

Continued reversal of the magnetizing force produces the hysteresis loop shown in Fig. 1, where the area under the loop is directly proportional to the energy required to magnetize the core.

INCREMENTAL INDUCTANCE: If an inductor is driven in one direction by DC, upon which is superimposed a small AC signal, the relationship between B and H is described by the minor hysteresis loop shown in Fig. 2. Under these conditions, it is the inductance that is offered to the superimposed AC signal that is of interest. This is called the incremental inductance, and the corresponding permeability, which is the slope of the B-H curve at the instantaneous point of operation, is the incremental permeability, $\Delta B/\Delta H$. As seen in both Fig. 1 and Fig. 2, the incremental permeability falls off rapidly as the flux density approaches B_{SAT} .

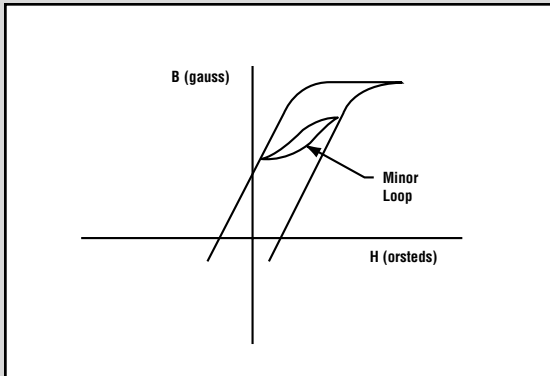


FIGURE 2. B-H Minor Loop

Elementary magnetic circuit analysis provides a relationship between the inductance, the magnetic properties of the core, and the physical configuration of the flux path as follows:

$$L = 0.4\pi N^2 K \cdot 10^{-8} \text{ Henrys} \quad (4)$$

where N is the number of turns, and K is a factor that is directly proportional to the permeability of the magnetic material, directly proportional to the cross

sectional area of the magnetic path and inversely proportional to the length of the path (including air-gap if present).

Since the incremental permeability changes as a function of operating point, it follows from equation (4) that the inductance also changes. In forward type converters, however, the minimum inductance is selected (by design) to be of such value as to ensure that the AC component of current through the inductor, referred to as the inductor ripple current, is normally only 10% to 20% of the average inductor current. This results in a relatively small minor B-H loop with a fairly constant slope, thus, the value of inductance can be considered constant over the complete AC current variation, as long as the average flux density is maintained sufficiently below B_{SAT} .

ENERGY STORAGE IN AN INDUCTOR: An alternative definition of inductance states that it is a measure of the ability of an inductor to store energy in a magnetic field. It is important to emphasize that an inductive device stores energy as opposed to dissipating energy. It is this property of inductors that makes them ideal for use in feed-forward type converters. The total energy input, W_L , in joules is directly proportional to the square of the final current in amperes, and the factor of proportionality is $L/2$, where L is in henrys. Expressed as an equation:

$$W_L = 1/2 LI_L^2 \text{ Joules} \quad (5)$$

Actually, there are energy losses which prevent 100% of the input energy from being stored and ultimately transferred to an output. The energy required to magnetize the core (referred to as hysteresis loss) and eddy current loss in the core, itself, make up the so-called core losses. Also contributing are the copper losses, i.e., energy dissipated in the coil as a result of the coils DC resistance and the frequency dependent skin-effect resistance.

CONTINUOUS VS. DISCONTINUOUS

CONVERTER OPERATION: A forward type converter is said to be operating in “continuous mode” as long as the current through the inductor remains above zero for the entire switching cycle. If, at minimum output current, the current through the inductor drops to and remains at zero during some portion of the inductor discharge cycle, the converter is being operated in “discontinuous mode.” Forward converters can be operated in either continuous or discontinuous mode; and by definition, L_{MIN} is that value of inductance, below which, the converter just becomes discontinuous at minimum output current.

CALCULATING MINIMUM INDUCTANCE FOR A BUCK TYPE CONVERTER

INDUCTOR RIPPLE CURRENT: Referring to Fig.’s 3 and 4, during T1 (SW_{ON}), the current through the inductor increases, and additional energy is stored in the inductor’s magnetic field in accordance with equation (5). During T2 (SW_{OFF}), the current decreases, and the added energy (minus the losses) is transferred to the load. The output voltage, V_O , is maintained at a steady preset value by virtue of the pulse width modulation (PWM) feedback loop. The variation in current through the inductor is

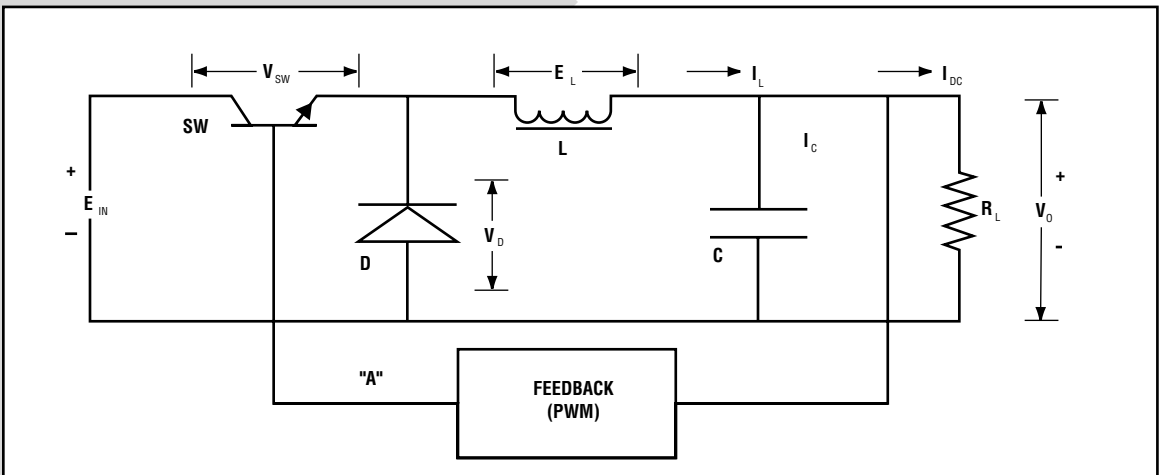
the ripple current. Since the load current, I_{DC} , is constant (for a given V_O and load resistance, R), it is apparent that the capacitor, C , is “absorbing” the variations in I_L , acting as a current sink for the inductor during the time interval, x-y, and a current source for the load during y-z. The larger the value of C , the lower will be the output ripple voltage.

SWITCH ON AND OFF TIMES: In order to determine a minimum required inductance value, it is necessary to calculate the period of time during which energy is either added to or transferred from the inductor. In addition the following circuit parameters must be known:

- a) Power supply switching frequency, F
- b) DC supply voltage, E_{IN}
- c) Output voltage, V_O
- d) Output current, I_{DC}

Referring to Fig. 4c, since both T1 and T2 are normally short compared to the time constant associated with L , the slopes of the current vs. time waveform can be considered straight lines. Thus, di/dt in equation (3) can be replaced by $\Delta I_L/T1$ during the “on” time and by $\Delta I_L/T2$ during the “off” time.

FIGURE 3. Buck Circuit



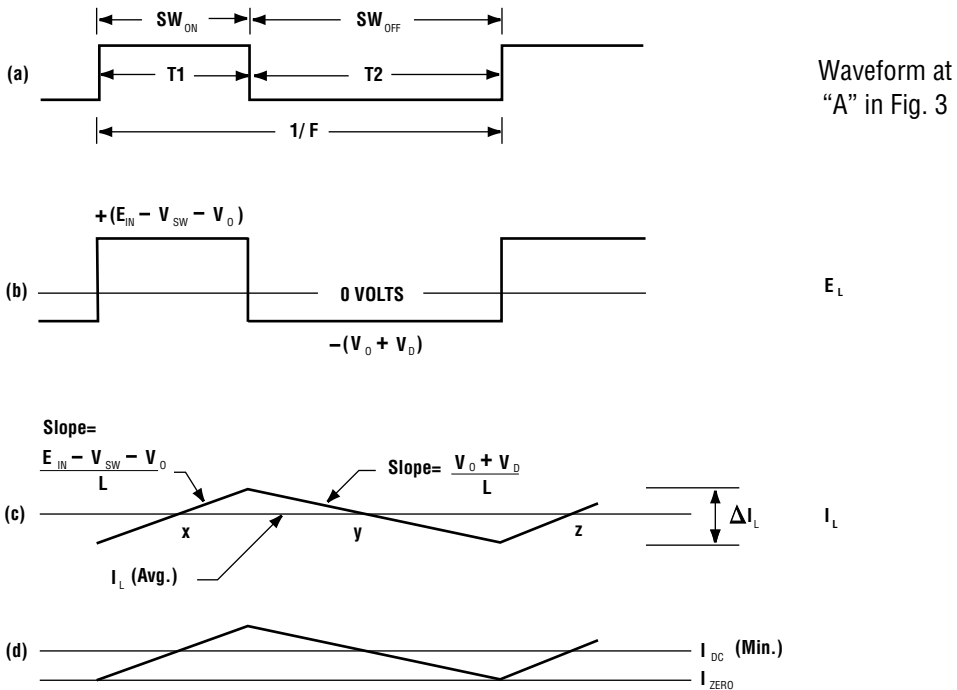


FIGURE 4. Buck Circuit Waveforms

During T1, equation (3) becomes:

$$E_{L1} = E_{IN} - V_O - V_{SW} = L\Delta I_L / T1 \quad (6)$$

and during T2:

$$E_{L2} = V_O + V_D = L\Delta I_L / T2 \quad (7)$$

Thus:

$$(E_{IN} - V_O - V_{SW}) T1 = (V_O + V_D) T2 \quad (8)$$

Equations (6) and (7) show that since ΔI_L is the same for both T1 and T2, inductance can be calculated simply by knowing ΔI_L plus the voltage across the inductor during the corresponding period. In this case the expression for T1 will be used. Solving equation (8) for T2 and adding T1 to both sides gives:

$$T1 = (V_O + 0.5) / FE_{IN} \quad (9)$$

where V_{SW} and V_D are assumed to be 0.5V and $F = 1 / (T1 + T2)$

DETERMINING INDUCTOR RIPPLE CURRENT:
Since F is constant, equation (9) shows that T1 (and consequently T2) is dependent only on the DC input and output voltages. Thus, for a given V_O , as long as E_{IN} remains unchanged, the peak-to-peak ripple current, ΔI_L , will remain constant regardless of the value of $I_{L(AVG)}$, which, for a Buck type converter, equals the output current. As seen in Fig. 4d, as I_{DC} decreases, a point is reached where the current through the inductor just reaches zero. A further decrease in I_{DC} results in "discontinuous operation" of the converter. Although it is possible to operate in discontinuous mode, such operation places more stringent criteria on the design of the power supply. For this reason ΔI_L is normally chosen to be no more than twice the minimum value of the current through the inductor - which in turn is normally chosen to be 10% to 20% of the maximum inductor current; and

for a Buck converter, this is equivalent to saying that the inductor ripple current is chosen to be twice I_{DC} (MIN), which is normally 10% to 20% of I_{DC} (MAX).

RIPPLE CURRENT VS. INDUCTOR SIZE AND TEMPERATURE: Since theoretically, I_{DC} (MIN) can be any % of I_{DC} , the 10% to 20% figure is a compromise as explained in the following: If the minimum output current is only slightly less than the maximum, then theoretically the inductor ripple current can be quite large. According to equation (3), a larger ΔI_L means a lower inductance value for a given ET. This means that for a given DC current through the inductor, the inductor itself could be physically smaller, which is, of course, desirable. However, if ΔI_L is large, then less energy can be stored for a comparable temperature rise due to increased core and $I_L^2 R$ losses, (where I_L is the RMS value of ΔI_L , and R is the resistance of the inductor). These losses could result in a temperature increase exceeding the allowable specification. Conversely, a very low ripple current requires a high inductance value, which implies a physically larger inductor. Therefore, a compromise must be made in the ripple current that maintains both the inductor size and temperature rise at acceptable levels.

CORE SATURATION: A further complication of a large AC ripple current is the fact that it produces a larger flux swing on the B-H loop of the inductor core. Fig. 5 shows that if the average current through the inductor is generating a core flux close to the knee of the B-H curve, then a large ripple current could cause the core to saturate during the positive peaks.

It is important to note that unlike the current through the inductor, which can have both a DC and an AC component, Faraday's Law states that the average voltage across an inductor over a complete cycle must be zero (see Fig. 4b).

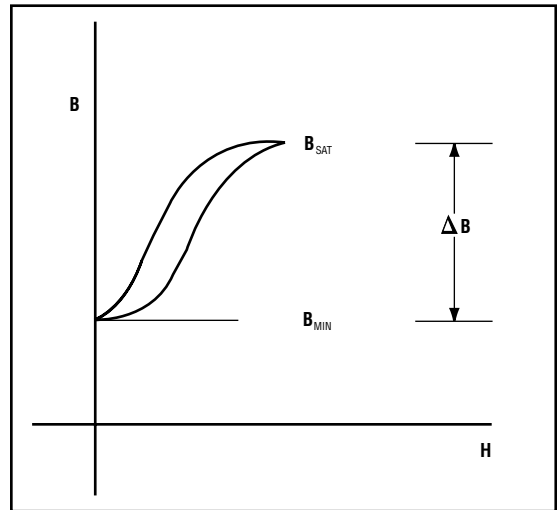


FIGURE 5. Non-Symmetrical Flux Swing

EQUATION FOR L_{MIN} : Fig. 4c shows that for a given inductor ripple current and a given output voltage, the largest value of L_{MIN} (required to ensure continuous converter operation at minimum I_{DC}) will occur when T2 is maximum and, consequently, when T1 is minimum. As seen from equation (9), T1 will be minimum when E_{IN} is maximum. Once T1 and ΔI_L are known, an expression for the minimum inductance can be derived using equation (6):

$$L_{MIN} = (E_{IN(MAX)} - V_O - 0.5) T1_{MIN} / \Delta I_L \quad (10)$$

VOLT-SECONDS DEFINED: The numerator of equation (10) is the volt-seconds of the inductor, commonly referred to simply as the “ET.” Most manufacturers of switching type magnetic components provide a maximum ET as well as a nominal inductance value for inductors designed for energy storage applications. Knowing these parameters, equation (10) can be used for a Buck converter to determine the peak AC current that the inductor can safely handle without core saturation.

Equation (8) and Fig. 4b show that the ET's during both T1 and T2 are equal. If this were not the case, the core would not “reset” during each cycle, i.e., the

amount of energy added during T1 would not equal the energy transferred during T2, and the core would quickly saturate.

BUCK CONVERTER DESIGN EXAMPLE: The following is a design of the minimum required inductance for a Buck type converter with the following specifications:

- Input Voltage+22V to +26V
- Output Voltage.....+5V
- Switching Frequency50kHz
- Max DC Output Current2.5 Amps
- Min DC Output Current0.5 Amps

Using the maximum input voltage, equation (9) gives the minimum on-time of the transistor switch as follows:

$$T1_{MIN} = (5+0.5) / 26 \cdot 50000 = 4.23 \mu\text{seconds}$$

Selecting a value of 1 amp for the inductor ripple current (twice the minimum value of DC output current), and substituting in equation (10) gives for L_{MIN} :

$$L_{MIN} = (26-5-0.5)(4.23 \cdot 10^{-6}) / 1 = 86.7 \mu\text{henrys}$$

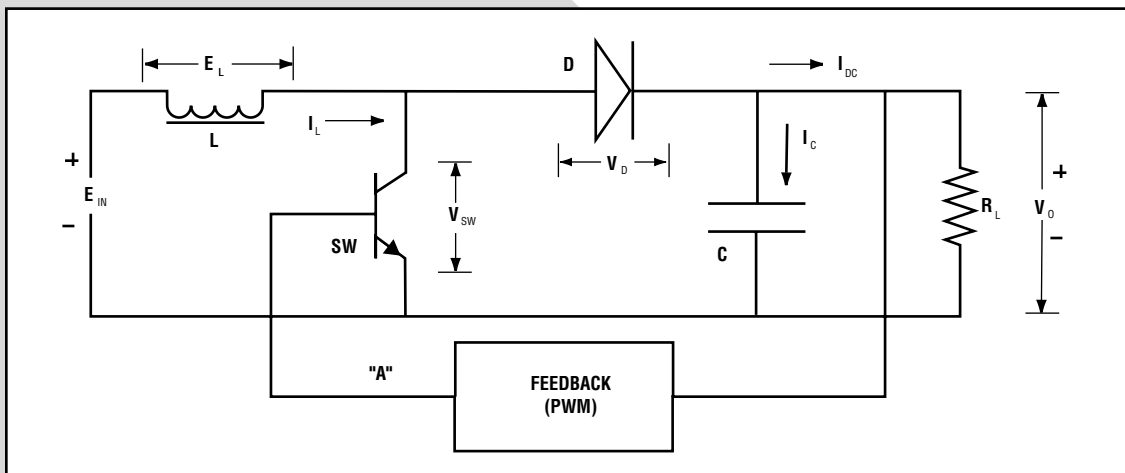
CALCULATING MINIMUM INDUCTANCE FOR A BOOST TYPE CONVERTER

CIRCUIT ANALYSIS: The analysis for the Boost converter is similar to that for the Buck with one basic difference. As seen in Fig. 6, the current through the inductor does not flow continuously to the output as in the Buck converter. Instead, during T1 (SW_{ON}), the output end of the inductor is switched (essentially) to ground, and diode, D, is reversed biased. Thus, during T1, the entire output current must be supplied by the output capacitor, C. During T2 (SW_{OFF}), the output end of the inductor swings positive, the diode becomes forward biased, and a positive output voltage is produced. The actual value of V_O is determined by the relative on/off times of SW. As seen in Fig. 7b, V_O must exceed E_{IN} in order for the average voltage across L to equal zero as required.

SWITCH ON AND OFF TIMES: Since the average voltage across the inductor must be zero, Fig. 7b gives:

$$(E_{IN} - V_{SW}) T1 = (V_O + V_D - E_{IN}) T2 \quad (11)$$

FIGURE 6. Boost Circuit



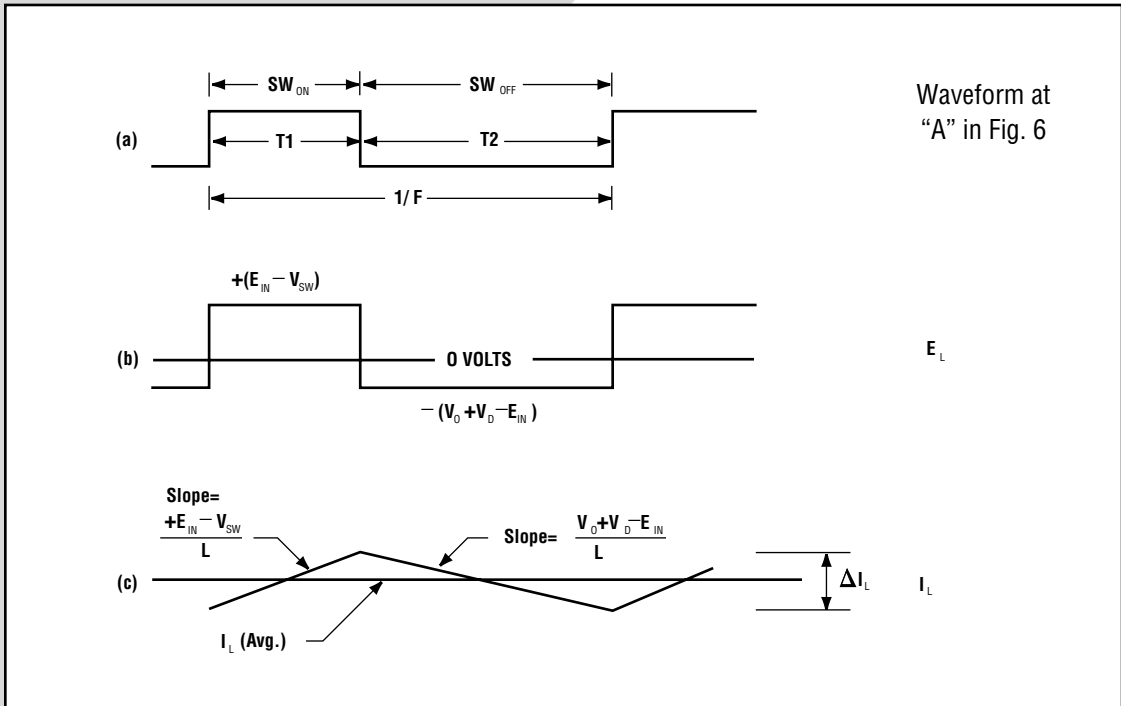


FIGURE 7. Boost Circuit Waveforms

Solving equation (11) for T2 and adding T1 to both sides gives:

$$T1 = (V_O - E_{IN} + 0.5) / FV_O \quad (12)$$

where V_{SW} and V_D are again assumed to be 0.5V

DETERMINING INDUCTOR RIPPLE CURRENT:

Since the average current through the inductor does not equal the DC output current (as in the Buck circuit), it is necessary to equate input power and output power to derive an expression for $I_{L(AVG)}$.

$$\text{Thus: } E_{IN}I_{L(AVG)} = 1.05 (V_O + 0.5) I_{DC} \quad (13)$$

where the 0.5 accounts for the diode drop, and the factor 1.05 compensates for typical core and circuit losses.

$$\text{And: } I_{L(AVG)} = 1.05 (V_O + 0.5) I_{DC} / E_{IN} \quad (14)$$

Assuming a ΔI_L at twice the minimum I_L , and a minimum I_L at P% of $I_{L(AVG)}$, gives for the inductor peak-to-peak ripple current, ΔI_L :

$$\Delta I_L = 2 (P / 100) I_{L(AVG)} \quad (15)$$

Since the largest L_{MIN} occurs when ΔI_L is minimum (i.e., the larger the L, the lower the ripple current), the maximum value of E_{IN} must be used in equation (14) when calculating $I_{L(AVG)}$.

The minimum DC output current corresponding to the ΔI_L in equation (15) can be found simply by solving equation (14) for I_{DC} using $\Delta I_L / 2$ as the value for $I_{L(AVG)}$ - again, using $E_{IN(MAX)}$ for the input voltage. Thus, the minimum output current that can be drawn from the power supply while just

maintaining continuous operation is:

$$I_{DC(MIN)} = \Delta I_L E_{IN(MAX)} / [2.1 (V_O + 0.5)] \quad (16)$$

Conversely, equation (16) can be used to solve for a ΔI_L corresponding to a specified $I_{DC(MIN)}$.

$$\text{Thus: } \Delta I_L = 2.1 (V_O + 0.5) I_{DC(MIN)} / E_{IN(MAX)} \quad (17)$$

EQUATION FOR L_{MIN} : The voltage across the inductor during T1 was seen to be:

$$E_{L1} = E_{IN} - V_{SW} \quad (18)$$

which, from equation (3) becomes:

$$E_{L1} = E_{IN} - V_{SW} = L \Delta I_L / T1 \quad (19)$$

As was the case for the Buck converter, the largest value of L_{MIN} will occur when T2 is maximum and T1 is minimum; and as before, T1 is minimum when E_{IN} is maximum, thus:

$$L_{MIN} = (E_{IN(MAX)} - 0.5) T1_{MIN} / \Delta I_L \quad (20)$$

BOOST CONVERTER DESIGN SAMPLES: The following are calculations of the minimum inductance for two Boost type converters:

Example Number 1:

Input Voltage+ 12V to + 15V

Output Voltage+ 24V

Switching Frequency50 kHz

Max DC Output Current1.5 Amps

Min DC Output CurrentNot Specified

Assume a maximum inductor ripple current of 12.5% of $I_{L(AVG)}$

$$\text{From (12): } T1_{MIN} = (24 - 15 + 0.5) / 50000 \cdot 24 = 7.92 \mu\text{seconds}$$

$$\text{From (14): } I_{L(AVG)} = 1.05 (24 + 0.5) 1.5 / 15 = 2.57 \text{ Amps}$$

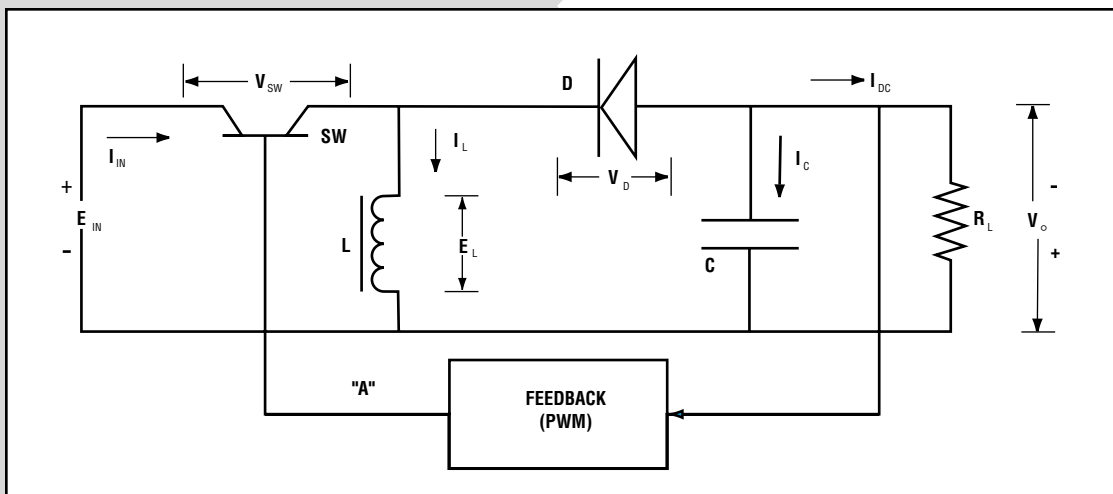
$$\text{From (15): } \Delta I_L = 2 \cdot 0.125 \cdot 2.57 = 0.643 \text{ Amps}$$

$$\text{From (20): } L_{MIN} = (15 - 0.5) (7.92 \cdot 10^{-6}) / 0.643 = 179 \mu\text{henrys}$$

The minimum DC output current corresponding to a ΔI_L of 0.643 Amps is:

$$\text{From (16): } I_{DC(MIN)} = 0.643 \cdot 15 / [2.1 (24 + 0.5)] = 0.19 \text{ Amps}$$

FIGURE 8. Buck-Boost Circuit



Example Number 2 - Same as example 1 except

$I_{DC(MIN)}$ is specified at 0.1 Amps:

$$\text{From (17): } \Delta I_L = (2.1 \cdot 0.1) (24 + 0.5) / 15 \\ = 0.343 \text{ Amps}$$

$$\text{From (20): } L_{MIN} = (15 - 0.5) (7.92 \cdot 10^{-6}) / 0.343 \\ = 335 \mu\text{henrys}$$

CALCULATING MINIMUM INDUCTANCE FOR A BUCK-BOOST CONVERTER

CIRCUIT ANALYSIS: The analysis for the Buck-Boost converter, shown in Fig. 8, is similar to that of the Boost in that $I_{L(AVG)}$ is determined by equating input and output power. However, the Buck-Boost is somewhat more complex because of the on/off characteristic of the input current, I_{IN} , as shown in Fig. 9c. During T1 (SW_{ON}), the input current charges the inductor, i.e., the inductor current changes from $I_{L(MIN)}$ to $I_{L(MAX)}$. During T2 (SW_{OFF}), the input current drops to zero and the inductor current

decreases back to $I_{L(MIN)}$. Since the average input current during T1 is simply $I_{L(AVG)}$, and during T2 is zero, the average input current over a complete period is:

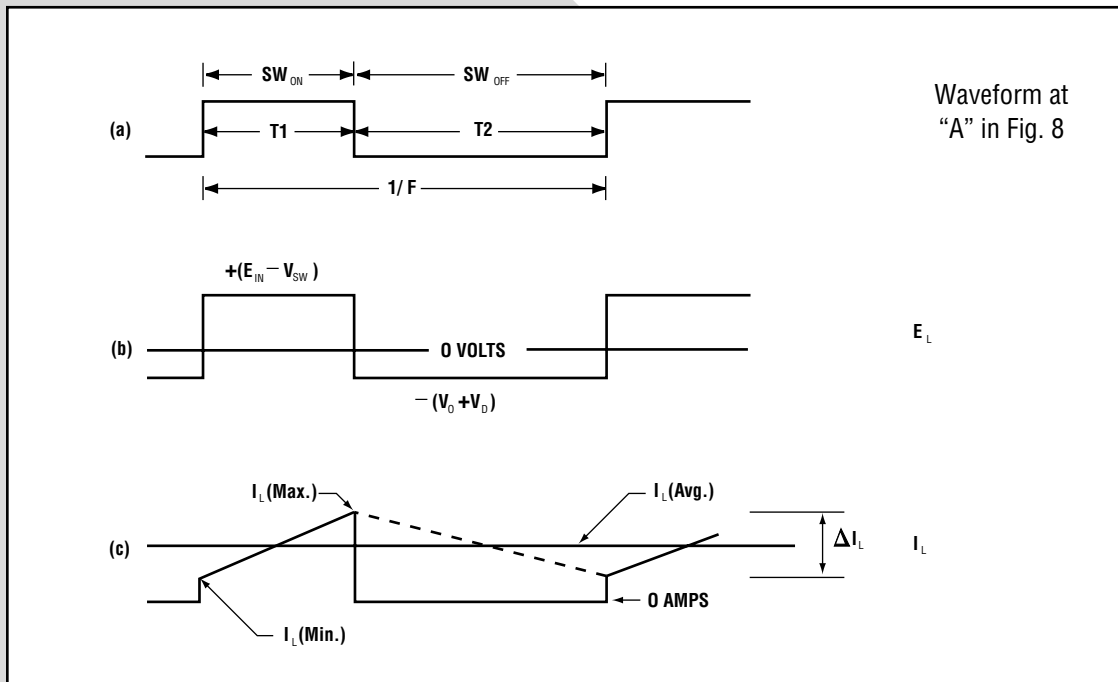
$$I_{IN(AVG)} = I_{L(AVG)} \cdot T1 / (T1 + T2) = F \cdot T1 \cdot I_{L(AVG)} \quad (21)$$

From Fig. 9b and equation (21) it is now possible to write an expression for Power-In equal to Power-Out which involves $I_{L(AVG)}$:

$$E_{IN} \cdot F \cdot T1 \cdot I_{L(AVG)} = 1.05 (V_O + 0.5) I_{DC} \quad (22)$$

where, again, the 0.5 accounts for the diode drop and the 1.05 compensates for typical core and circuit losses.

FIGURE 9. Buck-Boost Circuit Waveforms



Solving equation (22) for $I_{L(AVG)}$ gives:

$$I_{L(AVG)} = 1.05 (V_O + 0.5) I_{DC} / [E_{IN} \cdot F \cdot T1] \quad (23)$$

Referring again to Fig. 9b, it is obvious that V_O must be of opposite polarity to E_{IN} in order to satisfy the requirement that the average voltage across L be zero. Thus, the Buck-Boost circuit not only can step a voltage up or down (as its name implies), but it also provides polarity reversal.

SWITCH ON AND OFF TIMES: From Fig. 9b:

$$T1 (E_{IN} - V_{SW}) = T2 (V_O + V_D) \quad (24)$$

Solving equation (24) for T2 and adding T1 to both sides gives:

$$T1 = (V_O + 0.5) / [F (E_{IN} + V_O)] \quad (25)$$

DETERMINING INDUCTOR RIPPLE CURRENT:

The expression for the inductor ripple current for the Buck-Boost converter is the same as that for the Boost, namely:

$$\Delta I_L = 2 (P / 100) I_{L(AVG)} \quad (26)$$

The $I_{DC(MIN)}$ corresponding to ΔI_L in equation (26) is found (as before) by solving equation (23) for I_{DC} using $\Delta I_L / 2$ in place of $I_{L(AVG)}$.

$$I_{DC(MIN)} = \Delta I_L E_{IN(MAX)} / [2.1 (E_{IN(MAX)} + V_O)] \quad (27)$$

And, conversely, if $I_{DC(MIN)}$ is specified, equation (27) can be used to calculate the corresponding ΔI_L .

$$\Delta I_L = 2.1 (E_{IN(MAX)} + V_O) I_{DC(MIN)} / E_{IN(MAX)} \quad (28)$$

EQUATION FOR L_{MIN} : As seen from equation (25), T1 is minimum and T2 is maximum (which corresponds to the largest L_{MIN}) when E_{IN} is maximum, thus:

$$L_{MIN} = (E_{IN(MAX)} - 0.5) T1_{MIN} / \Delta I_L \quad (29)$$

Buck-Boost Converter Design Example: The following is a design of the minimum required inductance for a Buck-Boost converter with the following specifications:

Input Voltage+15V to +20V

Output Voltage-12V

Switching Frequency40kHz

Max DC Output Current.....0.75 Amp

Assume a minimum ripple current of 12.5% of $I_{L(AVG)}$

From (25): $T1 = 12.5 / 40000 (20 + 12) = 9.77$
 μ seconds

From (23): $I_{L(AVG)} = 1.05 \cdot 12.5 \cdot 0.75 / (20 \cdot 40000 \cdot 9.77 \cdot 10^{-6}) = 1.26$ Amps

From (26): $\Delta I_L = 2 \cdot 0.125 \cdot 1.26 = 0.315$ Amps

Thus, from (29): $L_{MIN} = (20 - 0.5) (9.77 \cdot 10^{-6}) / 0.315 = 605$ μ henrys.

